ALM - Risk Measurement

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Course Program

- Basic interest rate theory
- Interest rate risk management
- Stochastic term structure models
- `Risk measurement`
- Reinsurance and insurance-linked securities
- Mean-variance analysis for ALM

Contents of the chapter

- VaR and TailVaR.
- Simulation of stochastic evolution of the yield curve.
- Coherent risk measures
- Spectral risk measures
- Solvency II and the 200 year event

Risk measures

There are two main forms of risk measurement in ALM.

- Analysis of `sensitivity to changes` in financial variables:
  - Duration
  - Convexity
- Modeling a probability distribution and `simulation`:
  - Value at Risk (`VaR`)
  - Tail Value at Risk (`TailVar`)
  - Coherent risk measures
  - Spectral risk measures

Definition of a loss

- A random variable X is a `loss` if
  - $X > 0$ denotes a loss
  - $X < 0$ denotes a profit.
- The probability distribution F of X is a `loss distribution`. In most applications
  - $\text{Loss} X = \text{Actual Cost} - \text{Expected Cost}$, or
  - $\text{Loss} X = \text{Expected Income} - \text{Actual Income}$

Value at Risk (VaR)

- For a loss distribution F, VaR at confidence level α is defined as the α-percentile:

$$\mathrm{VaR}_{\alpha}(F) = F^{-1}(\alpha)$$

- VaR is the loss level that will not be exceeded with a given probability. For example, α could be $0.90,\ 0.095,\ 0.99,\ 0.995.$
- Var involves two arbitrary parameters:
  - the `confidence level` α
  - the `holding period` (day, month, or until final run-off)

Value at Risk (VaR) - Observations

- Started by JP Morgan for one-day measurements - RiskMetrics https://www.msci.com/documents/10199/5915b101-4206-4ba0-aee2-3449d5c7e95a
- It is a `single measure` that applies to all types of losses and aggregates.
- The requirement of a probability implies that the losses are `normal`.
- VaR `omits the severity` of events that surpass the confidence level.

Calculating Value at Risk - I

- Historical observations:
  - Easy
  - Realistic, if enough observations are available
- Historical observations with resampling:
  - Easy
  - Realistic, if level of confidence is realistic itself
  - `Does not go beyond observed values`

Calculating Value at Risk - II

- Fitted distributions and simulation:
  - More complicated, possible model error.
  - Allows for a tail beyond the observed.
  - Not necessarily better than the previous, within the normal range of outcomes.

Criticism of VaR

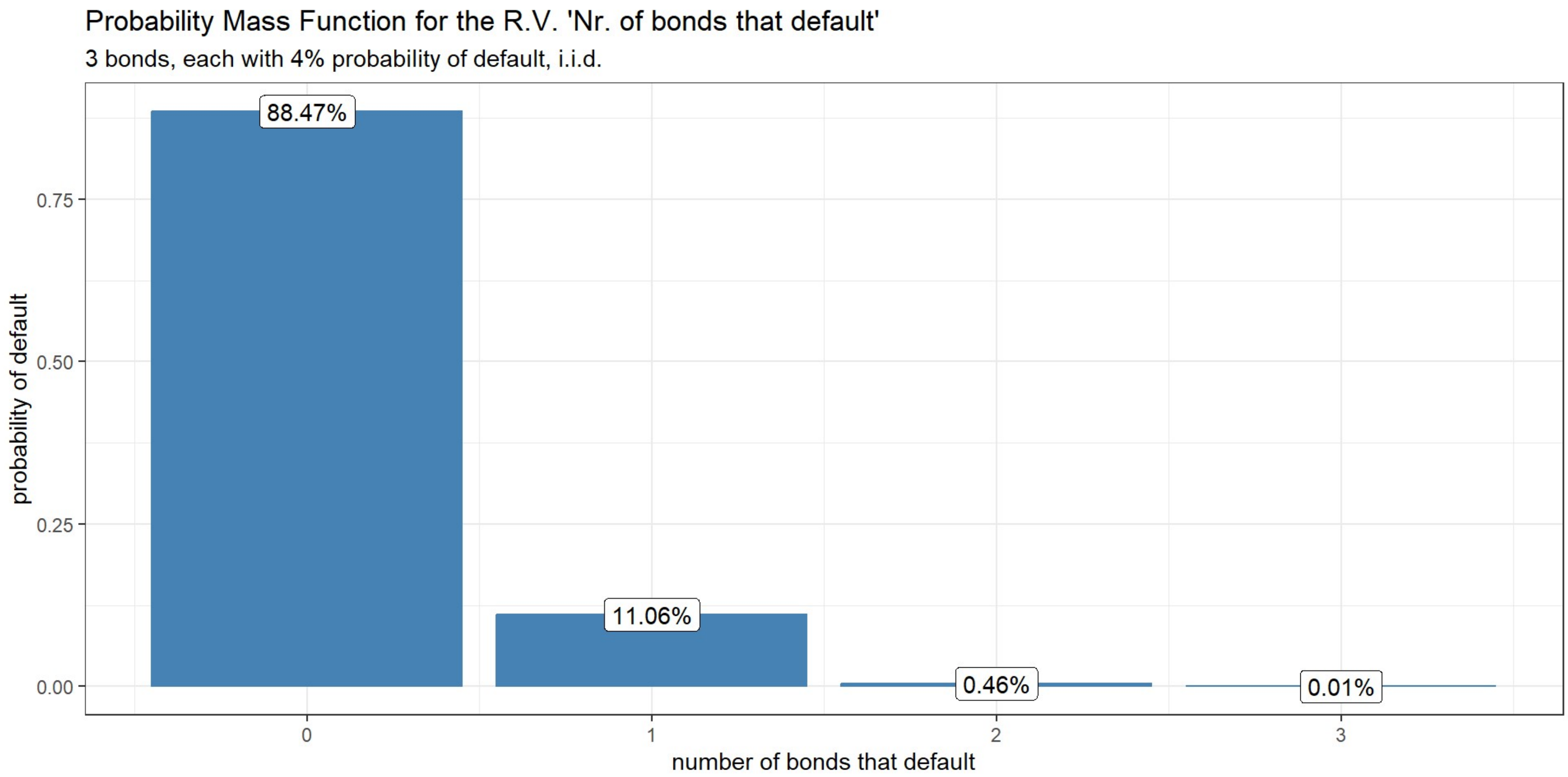
- VaR does not tell us the `severity` of events beyond the confidence level.
- Two portfolios with the same VaR may have different tail distributions.
- VaR does not necessarily recognize diversification benefits.
- "Perverse incentives", "discourage diversification" and so on.
- `Response`: Tail Value at Risk (TailVaR)

Example VaR Diversification I

- You have a portfolio of three `100€ bonds`, each with a `4%` probability of default, independent from each other.
- Each bond follows a `Bernoulli` distribution, so the random variable **X** - *nr. of bonds that default* - follows a `Binomial`.
- Inspection of its probability mass function leads to one conclusion:

95% VaR tells us that it is less risky to invest all in only 1 bond!

Example VaR Diversification II



Expected Tail Loss - TailVaR

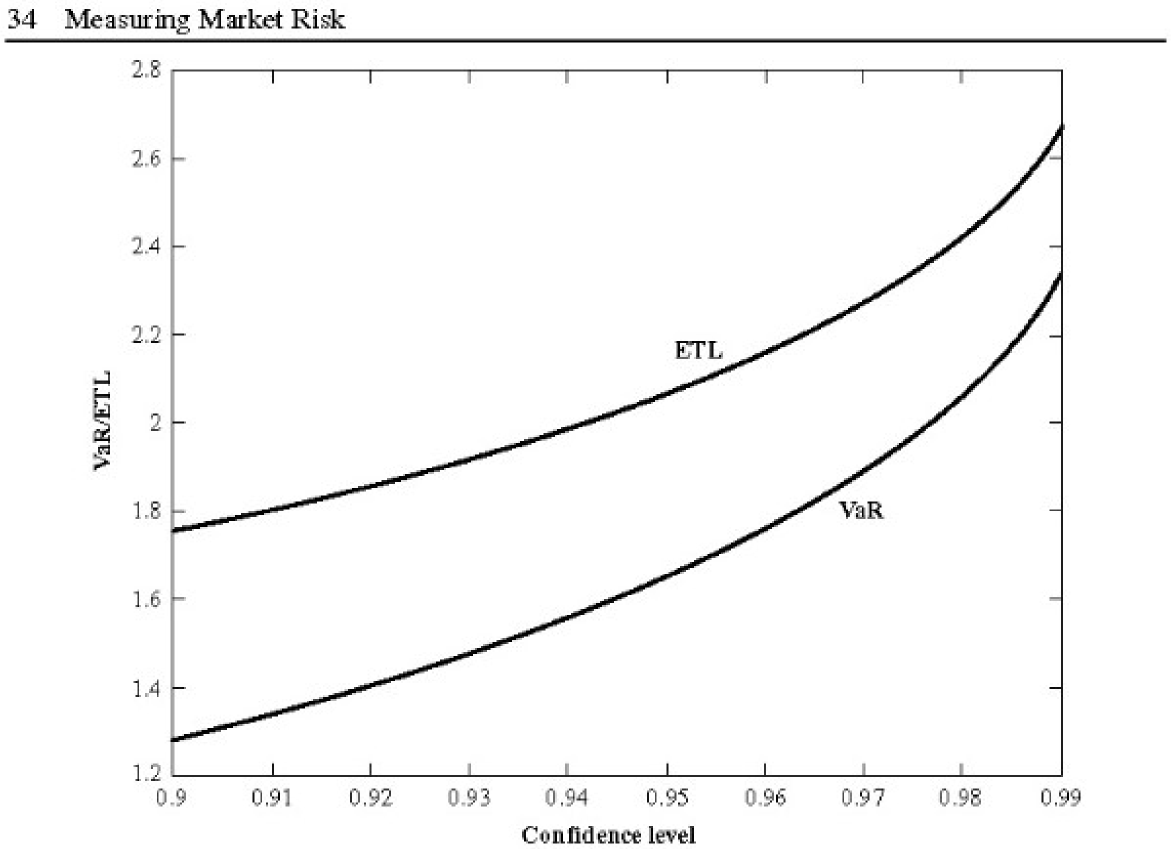
- For a loss distribution F, TailVaR at confidence level α is defined as

$$\mathrm{TailVaR}_{\alpha}(F) = \frac{1}{1-\alpha}\int_{\alpha}^{1} \mathrm{VaR}_{s}(F)ds$$

- TailVaR is the average of VaR above the confidence level α.

VaR and TailVaR Illustration

- TailVaR compared with VaR vs. confidence level (Normal dist.)



TailVaR - Observations

- TailVaR measures the `entire tail` beyond the VaR percentile.
- TailVaR is `indifferent to the size of losses` that exceed VaR.
- TailVaR is not the only alternative to VaR.
- Its choice of confidence level and holding period is also `arbitrary`.
- Quantifying extreme tail losses and their probability is difficult.

Coherent risk measures

- A risk measure ρ is a functional of a loss distribution F, or alternatively a loss random variable $X \sim F$.
- The risk measure is `coherent` if it satisfies:
  - Monotonicity: $X \geq Y \Rightarrow \rho(X) \geq \rho(Y)$
  - Subadditivity: $\rho(X+Y) \leq \rho(X) + \rho(Y)$
  - Positive homogeneity: $\rho(aX) = a\rho(X)$ if $a > 0$ is a constant
  - Tranlation invariance: $\rho(X+a) = \rho(X) + a$ for a constant a.
- `TailVaR is coherent, while VaR is not`.

Spectral risk measures I

- A `spectral risk measure` is defined as a weighted average of VaR at different levels.
- The weight increases with the severity of the loss:

$$\rho(F) = \int_0^1 \psi(s) F^{-1}(s) ds = \int_0^1 \psi(s) VaR_s ds$$

where $\psi(s) \geq 0, \int_0^1 \psi(s) ds = 1$ and $\psi(\cdot)$ is non decreasing (i.e. larger losses matter more).

Spectral risk measures I

- TailVaR is a spectral risk measure with

$$\psi(s) = \frac{1}{1-\alpha} I(s \geq \alpha)$$

  where I is a step function - **Homework!**
- One can prove that `spectral risk measures are coherent`.
- Good video introduction https://ryanoconnellfinance.com/product/expected-shortfall-value-at-risk-calculator-in-excel/

Solvency II and the 99.5% VaR

- The SCR and capital charges are based on 99.5% VaR. Why?
  - CEIOPS originally proposed 99% VaR as the basis of SCR.
  - The academic community objected that VaR was bad and that TailVaR should be used. They are right, of course.
  - CEIOPS put out a proposal to use 99% TailVaR.
  - The insurance industry and regulators objected that TailVaR was too complicated. They are right, of course.
  - As a compromise, CEIOPS suggested to use 99.5% VaR as a proxy for 99% TailVaR.